9). Use Kuhn- Tucker conditions to solve the NLPP

$$\max.\ z = 8x_1 + 10x_2 - x_1^2 - x_2^2$$

s.t.

$$3x_1 + 2x_2 \leq 6$$

$$x_1, x_2 \geq 0$$

Solution.

Here, $f(X) = 8x_1 + 10x_2 - x_1^2 - x_2^2$

$h(X) = 3x_1 + 2x_2 - 6 \leq 0$

where, $X = (x_1, x_2)$

Hessian matrix is

$$H^B = \begin{bmatrix} \frac{\partial^2 f}{\partial x_1^2} & \frac{\partial^2 f}{\partial x_1 \partial x_2} \\ \frac{\partial^2 f}{\partial x_2 \partial x_1} & \frac{\partial^2 f}{\partial x_2^2} \end{bmatrix}$$

$$= \begin{bmatrix} -2 & 0 \\ 0 & -2 \end{bmatrix}$$

∴ The principal minor are

$\Delta_1 = -2$

$\Delta_2 = 4$

which are of alternate signs

Hence, $f(x)$ is concave function ~~f of~~ of x_1 and x_2

Also $h(X) = 3x_1 + 2x_2 - 6 \leq 0$
is convex function
and hence Kuhn-Tucker Necessary condition for maximum of $f(X)$ are also sufficient condition

$\therefore$ The necessary condition for existence of maximum value of $f(X)$ are

$$\frac{\partial f}{\partial x_1} - \lambda \frac{\partial h}{\partial x_1} = 0$$

$$\Rightarrow 8 - 2x_1 - 3\lambda = 0 \quad \text{--- (1)}$$

$$\frac{\partial f}{\partial x_2} - \lambda \frac{\partial h}{\partial x_2} = 0$$

$$\Rightarrow 10 - 2x_2 - 2\lambda = 0 \quad \text{--- (2)}$$

and $\lambda h(X) = 0$

$$\lambda(3x_1 + 2x_2 - 6) = 0 \quad \text{--- (3)}$$

Also, $h(X) \leq 0$

$$\Rightarrow 3x_1 + 2x_2 - 6 \leq 0 \quad \text{--- (4)}$$

$$\lambda \geq 0 \quad \text{--- (5)}$$

where, λ is the Lagrangian multiplier

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eq^n (5) $\Rightarrow$ either $\lambda = 0$ or $\lambda > 0$

If $\lambda = 0$ then eq^n (1) $\Rightarrow$ $x_1 = 4$

and eq^n (2) $\Rightarrow$ $x_2 = 5$

Therefore (4) $\Rightarrow$ $12 + 10 - 6 \leq 0$

which is absurd.

(i.e. $x_1 = 4$ & $x_2 = 5$ does not satisfy (4))

$\therefore \lambda \neq 0$

$\Rightarrow \lambda > 0$

(3) $\Rightarrow 3x_1 + 2x_2 - 6 = 0$ —— (6)

from (1), $2x_1 = 8 - 3\lambda$

$\Rightarrow x_1 = \frac{1}{2}(8 - 3\lambda)$

from (2), $2x_2 = 10 - 2\lambda$

$\Rightarrow x_2 = 5 - \lambda.$

Substituting in (6)

$\frac{3}{2}(8 - 3\lambda) + 2(5 - \lambda) - 6 = 0$

$\Rightarrow \lambda = \frac{32}{13}.$

$\therefore \quad x_1 = \frac{1}{2}\left(8 - 3 \times \frac{32}{13}\right) = \frac{8}{2 \times 13}$

$\therefore \quad \boxed{x_1 = \frac{4}{13}}$

$x_2 = 5 - \lambda$

$= 5 - \frac{32}{13}$

$\therefore \quad \boxed{x_2 = \frac{33}{13}}$

$\therefore$ stationary point $(x_1, x_2) = \left(\frac{4}{13}, \frac{33}{13}\right)$

max. $z = 8 \times \frac{4}{13} + 10 \times \frac{33}{13} - \left(\frac{4}{13}\right)^2 - \left(\frac{33}{13}\right)^2$

$= \frac{3601}{169}$

$= 21.307$